The "Myth" of the Minimum SAR Antenna Area Constraint

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Abstract

A design constraint traceable to the early days of spaceborne Synthetic Aperture Radar (SAR) is known as the minimum antenna area constraint for SAR. It specifies, as the name suggests, a minimum area for SAR antennas, to optimize performance. In this paper, it is confirmed that this constraint strictly applies *only* to the case where both the best possible resolution and the widest possible swath are the design goals. SAR antennas with smaller areas than the constraint allows are shown to be possible, increasing the trade space for SAR system design while still achieving excellent performance. The result rests on three insights into spaceborne SAR design that have each been implemented successfully: that the pulse repetition frequency can be smaller than the nominal Doppler bandwidth; that the processing bandwidth can be selected in the system design; and that the range swath for which data is recorded can be less than the illuminated swath. Smaller SAR antennas than that specified by the minimum area constraint have been used on a number of successful SAR missions to date, and should permit further, lower-cost SAR missions in the future.

trade off

Keywords: Synthetic Aperture Radar, Antenna Area

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I. INTRODUCTION

The design of antennas for Synthetic Aperture Radars has received a great deal of attention in the literature on the subject. Most antennas for spaceborne SARs are rather large in size, with a range of 9 to 15 meters being fairly typical for the antenna length (azimuth dimension), and somewhere between 10 and 20 times the wavelength for the antenna height (elevation dimension). Since spaceborne SARs flown to date have had wavelengths in the range 3 cm to 24 cm, this means that the antenna heights are fairly significant. Spaceborne SAR antennas are thus some of the largest size structures flown in space. Even when the antenna is folded for launch, their sheer size and mass means that they often need to be accommodated in a larger size launch vehicle shroud. Clearly, if smaller SAR antennas were possible, they might fit on missions which use a smaller launch vehicle, and which therefore cost less.

The SAR designer has many parameters to select in specifying a SAR system design, one of which is the antenna size (height and width). A constraint that is often used by SAR designers to help select these parameters is known as the Minimum SAR Antenna Area Constraint. This constraint states that antennas used in SAR systems must have a certain minimum area for the design to be viable. It is derived in many of the standard texts on the subject (e.g. [1], [2], [3] and [4]) via a thorough treatment of a special case of SAR design, for which the best possible resolution and the widest possible swath are the design goals. It is clear in the derivation given in [1], for example, that the constraint in question applies only "for realization of full resolution SAR". In this paper, this constraint is examined and shown to apply only in the special case referred to. A more general treatment is also offered, in which it is shown that smaller SAR antennas are practicable and offer the SAR system designer a greater degree of freedom in system design.

The first suggested use of an antenna with area smaller than the "minimum SAR antenna area constraint" provides for can probably be attributed to W. T. K. Johnson (JPL) who used this approach in his design of the Magellan SAR system [5], when imaging at larger incidence angles. Bill Johnson is also most likely responsible for the recognition that a PRF smaller than the nominal Doppler bandwidth could be used in his design for the Magellan SAR system. He later used the same approach in the design of the Titan Radar Mapper on the Cassini mission [6] which is en route to Saturn at the present date. The first actual use of a SAR antenna with area smaller than the "minimum antenna area constraint" was by R. Jordan and J. Curlander during the SIR-B mission, when the antenna, which was optimized for smaller look angles, was pointed out to 60 degrees off-nadir, for which it was not optimum. PRFs less than the Doppler bandwidth were also first used in the multi-angle SIR-B mission by J. Curlander and R. Jordan because the performance requirements for transmit interference, nadir interference and ambiguity requirements could not be met with any PRF at shallow look angles. For those data, the processing bandwidth was reduced in the ground-based processor and the resulting images were acceptable. R. Jordan (JPL) also used a small antenna in the design of the outboard antenna for the Shuttle Radar Topography Mission (SRTM), which is scheduled to fly in 1999. The insight into the use of a smaller processing bandwidth to avoid ambiguities in spaceborne SAR at the expense of degraded azimuth resolution is attributable to J. Curlander, who proposed this approach for SIR-B and for the multi-mode SIR-C radar instrument. It is also clear from one of the earliest publications on SAR ambiguities [7] that this is a viable approach.

In this paper, section II addresses the theory behind the derivation of the "minimum antenna area constraint" and shows that it really only applies in the special case referred to above, and not to SAR antennas in general. Section III gives examples of SAR antenna used in a selection of spaceborne SAR missions, and shows that in several cases they are smaller than the minimum antenna area constraint as applied in the literature would allow. Section IV discusses possible reasons why this constraint has been applied in the past, and the possibility of a broader class of missions in the future using smaller SAR antennas.

II. THEORY

The geometry under consideration is shown in Figure 1. A planar SAR antenna of length L_a, and height W_a, traveling along a straight line trajectory at speed V, is pointed in a side-looking direction, perpendicular to the flight track, so that it illuminates a swath on the ground of width $W_{g(max)}$. This illuminated swath width is determined by the beamwidth of the antenna in the elevation plane and the geometry of the situation, as follows:

$$W_{g(max)} \approx \frac{\theta_{el} R_m}{\cos \eta} = \frac{\lambda R_m}{W_a \cos \eta} \tag{1}$$

where the well-known expression for the 3-dB beamwidth of a planar array, i.e. $\theta_{el} = \lambda/W_a$, has been used. $W_{g(max)}$ represents the widest possible swath in ground range (or cross-track) for which data can be collected, given an antenna of a particular size and a certain illumination geometry.

Another result well-known to SAR designers is the limiting resolution in the azimuth (or along-track) dimension, given by:

$$\delta x \geq L_a / 2 \tag{2}$$

which simply states that the best possible azimuth resolution that can be achieved for a non-squinting, side-looking SAR with antenna of length L_a, is half that antenna length.

Because SARs are pulsed radar systems, the SAR designer's task is complicated by the need to consider ambiguous returns in both the azimuth and range dimensions. One basic requirement, adapting the arguments given in [1], is that the time of reception of the earliest possible echo from any point within the *desired* swath due to a particular pulse transmission must be later than the time of reception of the last possible echo from any other point within the *illuminated* swath due to transmission of the previous pulse. This avoids ambiguous returns in range from the main lobe of the antenna (in elevation) occurring within the desired swath. From Figure 1, this means that:

$$2R_4/c < 2R_2/c + IPP \tag{3}$$

where R_2 and R_4 are the near range limit of the desired swath in slant range and the far range limit of the illuminated swath respectively, and the interpulse period (IPP) is the inverse of the Pulse Repetition

Frequency (PRF), i.e. IPP = 1/PRF. Note that (3) assumes that the transmitted pulse length is significantly smaller than either of the two path lengths. It is straightforward to incorporate the pulse length into the expression if this is not the case.

Given (3), the width of the desired swath in slant range, W_s, is bounded by:

$$W_s \le \left(R_4 - R_2\right) < c\ IPP / 2 = c / (2\ PRF) \tag{4}$$

The case $R_1 = R_2$ and $R_3 = R_4$ is of interest, since the desired swath and the illuminated swath are the same. Note that the *desired* swath need not be as large as the widest possible swath, i.e. the *illuminated* swath. Thus the radar designer can choose to record data from a swath smaller than that illuminated on the ground and need only consider range ambiguities which impact the *desired* swath. The near range limit of the desired swath can be anywhere within the bounds of the illuminated swath. The far range limit of the desired swath can be anywhere between R_2 and R_4. Equation (4) is often expressed as an upper bound on the PRF, i.e.,

$$PRF < c / 2\left(R_4 - R_2\right) \tag{5}$$

Thus the smaller the distance between R_2 and R_4, the larger the PRF is allowed to be. For a desired swath width smaller than the illuminated swath, the optimum would be to select $R_1 < R_2$ and $R_3 = R_4$.

In the azimuth dimension, again after [1], the requirement is to measure Doppler frequency unambiguously over the range of frequencies needed to achieve resolution δx. This paces a lower bound on the PRF given by:

$$PRF > V/\delta x \tag{6}$$

In practice the PRF must be significantly greater than this lower bound to avoid aliasing within the processing bandwidth ($V/\delta x$) required to achieve the needed azimuth resolution. In the limit provided when the best possible resolution is required, as in equation (2), this lower bound becomes:

$$PRF > 2V/L_a \tag{7}$$

which states that the PRF in this case should be greater than the range of Doppler frequencies within the bounds of the area illuminated by the physical antenna in azimuth, which is the Doppler bandwidth for that length of antenna. Note that, for a desired resolution which is worse than the theoretical best possible, equation (6) allows the radar designer to select a PRF which is *smaller than* the Doppler bandwidth associated with the given length of the antenna. Also, again from [1], the azimuth ambiguities need only be evaluated over the processing bandwidth required to achieve the needed azimuth resolution, not over the entire range of frequencies which the PRF spans.

Combining the constraints given in (4) and (6) yields:

$$W_s < \frac{c}{2\,\mathrm{PRF}} < \frac{c}{2\,V}\delta x \tag{8}$$

which, as noted in [1], requires that the swath width W_S decrease as the azimuth resolution δx improves (i.e. becomes smaller). Rearranging (8), the relationship between (slant range) swath width and (azimuth) resolution can be more clearly seen,

$$\frac{W_s}{\delta x} < \frac{c}{2\,V} \tag{9}$$

which is a well-known result ([1], [5]). For Low Earth Orbit satellites, c/2V is nearly constant (at 20,000). For airborne systems, c/2V is typically in the range 300,000 to 750,000 and satisfying the constraint given in (9) is rarely a problem.

The swath width in slant range can be related to the swath width in ground range via the nominal relation:

$$W_s = W_g \sin \eta \tag{10}$$

[which easily generalizable to the case for wide swath SARs, for which η varies significantly across the swath.] Using equations (1) and (2), combined with (10), in equation (9), the constraint for the case when both the best possible resolution and the widest possible swath are required, can be obtained:

$$\frac{W_{s\,(max)}}{\delta x_{(min)}} = 2\,\frac{\lambda\, R_m}{W_a}\,\frac{\tan\eta}{L_a} < \frac{c}{2V} \tag{11}$$

So the antenna area is restricted in this case by:

$$A_a = W_a\, L_a > \frac{4\, V\lambda\, R_m}{c}\,\tan\eta \tag{12}$$

which is a form of the commonly used minimum antenna area constraint for SARs. SAR system designers often introduce an additional design margin on top of this, so that the actual area of the antenna is given by:

$$A_a = \frac{K\, 4\, V\lambda\, R_m}{c}\,\tan\eta \tag{13}$$

where K is in the range 1 to 3.

As is clear from the above, equations (12) and (13) only apply to a special case, which is when the radar designer seeks to achieve both the best possible resolution and the widest possible swath at the same time. The fundamental constraint is actually given in equation (9), which places a limit on the ratio of the swath width versus azimuth resolution that really only depends on the platform speed V. It is often the case that the radar designer is given requirements to achieve a swath width W_s smaller than the best possible, and an azimuth resolution δx which is larger than the theoretical limit. In this situation,

$$\frac{W_s}{\delta x} \le \frac{W_{s\,(max)}}{\delta x_{(min)}} = 2\,\frac{\lambda\, R_m}{W_a}\,\frac{\tan\eta}{L_a} \tag{14}$$

alternatively,

$$A_a = W_a\, L_a \le \frac{2\,\lambda\, R_m}{W_s}\,\tan\eta\;\delta x \tag{15}$$

which says that in general there is actually an *upper* bound or *maximum* for the antenna area, depending on the size of the swath and the desired azimuth resolution. It is straightforward to show, however, by comparing the right-hand sides of expression (12) and (15) that this upper bound on antenna area is always greater than or equal to the lower bound expressed in (12) for all realizable cases.

III. EXAMPLES

It is easy to show that using smaller antennas than specified by the minimum area constraint has been used successfully by a number of very successful SAR missions as demonstrated in the table below:

Mission	Lambda (m)	Platform speed (ms^{-1})	Altitude (km)	Inc. angle (deg)	Minimum Area K = 1	Minimum Area K = 2.6	Actual area (m^2)
SEASAT	0.24	7500	800	23	8.85	23.02	23
ERS-1/2	0.056	7500	785	23	2.03	5.27	10
Radarsat (θ_i = 20°)	0.056	7500	802	20	1.74	4.52	22.5
Radarsat (θ_i = 60°)	0.056	7500	802	60	15.55	40.44	22.5
Magellan (θ_i = 17°)	0.126	4000	2100	17	4.51	11.73	8.6
Magellan (θ_i = 47°)	0.126	8500	290	47	6.51	16.93	8.6
SRTM	0.056	7500	235	30	0.88	2.28	6.4
	0.056	7500	235	60	4.56	11.85	6.4
Cassini Radar	0.02	5500	4000	20	2.27	5.91	3.3
	0.02	5500	1000	40	1.61	4.18	3.3
Venera	0.08	5000	2000	7	1.32	3.43	8.4
	0.08	6000	1000	17	2.05	5.32	8.4
SIR-B (θ_L = 60°)	0.234	7500	354	66	45.72	118.88	23.3
SIR-C (θ_L = 63°)	0.234	7500	225	67	31.73	82.50	33.6
P-Band SAR	0.68	7500	600	23	18.81	48.91	18.1

Table 1: A comparison of the actual antenna area with the "minimum antenna area" for a selection of spaceborne SARs using two values of the margin factor K (K = 1 and K = 2.6).

Application of the "minimum antenna area constraint" with a margin of K=2.6 (as used for the Seasat SAR) would thus have precluded the use of the antennas implemented on the Magellan, Titan Radar Mapper, SIR-B, SIR-C and SRTM missions. Application of the "minimum antenna area constraint" with a margin of K=1 would have precluded the use of the antenna implemented on the multi-angle SIR-B mission at higher incidence angles.

The P-Band SAR referred to in Table 1 is a proposed Earth-orbiting SAR, with a modest design goal of 50 meter (1-look) resolution and a 50 km swath width.

IV. DISCUSSION

The derivation in section II and the examples presented in section III show that there is no need to constrain SAR antennas to be a certain minimum area. In particular, when designing a SAR system which does not have to achieve both the best possible resolution and the best possible swath width at the same time the SAR system designer is free to choose to use a smaller antenna than would be the case for a SAR optimized to achieve these goals. This has significant impact on the design of multi-mode SARs, e.g. NASA's proposed LightSAR instrument [6], which may be optimized for one mode but not another, and in the design of moderate resolution SARs, which may take advantage of non-planar antennas, and other antennas which are not optimized for SAR performance but which may be more cost effective. An excellent example of the latter was the Magellan SAR design [7], which took an existing 3.7 m diameter parabolic reflector antenna designed for communications and not optimized for SAR data collection, and successfully imaged 97% of the surface of Venus at 100-300 m resolution.

This does not mean that SAR antennas can be arbitrarily small in size. The size of the antenna has significant impact on the gain and therefore on the signal-to-noise ratio which must be taken into account. The analysis presented in this paper is no substitute for a rigorous treatment of the calculation of range and azimuth ambiguity levels, which must be factored in by the designer. The exact form of the antenna pattern and other radar parameters such as range, PRF, processing bandwidth, and the radar backscatter as a function of incidence angle must all be incorporated into such a calculation.

The result presented in this paper regarding the minimum antenna area rests on three key insights. The first is that the PRF need not be greater than the nominal Doppler bandwidth, as recognized by Bill Johnson in the design of the Magellan SAR, and as stated in [1]. In practice, to achieve reasonable azimuth ambiguity performance over even a small processing bandwidth, the PRF can not usually fall below about 75% of the nominal Doppler bandwidth. The second insight is that the swath for which data is actually recorded need not be as large as the illuminated swath, which is easier to incorporate into any SAR system design. The third insight is that the processing bandwidth in azimuth is a choice open to the SAR designer, provided it is less than the PRF, and delivers performance compatible with the data requirements.

Given the above, the question arises: why is the SAR design approach described in this paper not widely presented in SAR system design manuscripts and course notes and more widely embraced by the SAR design community? Several reasons are suggested for this:

1. The result is not widely known, hence the need for this paper
2. In earlier spaceborne SAR system designs, conservatism in the antenna design was highly desirable, because of uncertainties in the antenna beam pattern on deployment. In the case of ERS-1/2, the antenna area is especially large to allow for beam shaping in elevation.

3. Conservatism in the choice of PRF was also desirable in earlier SAR system designs because of large uncertainties in platform, and therefore antenna, pointing
4. Most earlier SAR systems were designed to optimize performance for a single mode of operation, or when more than one mode was planned, for the most challenging mode.
5. Most earlier spaceborne SAR systems had the design goal of achieving the best possible azimuth resolution, often so that more looks could be taken in azimuth at a somewhat degraded resolution. This precluded the use of a PRF lower than the nominal Doppler bandwidth.
6. Most spaceborne SAR systems were designed to achieve the best possible Signal-to-Noise Ratio. This precluded the adoption of a data swath in range which was significantly narrower than the illuminated swath because of the resulting decrease in antenna gain across the imaged area.

Current antenna technology and spacecraft pointing capabilities are significantly improved in comparison to the days of SEASAT for example, lessening the need for conservatism as expressed in 2 and 3. The earth science community seems to desire more modest (e.g. 100-250 m) resolution for future measurements, which eases the issue addressed in 5.

Optimizing SAR performance over all modes in some of the Earth-orbiting SARs planned for the next decade such as LightSAR could mean that the resulting antenna would be so large in area that it would have to be accommodated in a costly, large capacity launch shroud. Using smaller antennas in SAR system designs means that future SAR missions can cost less, but will still be able to deliver data to meet a wide variety of science needs. This widens the scope of possible SAR missions enormously.

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Figures

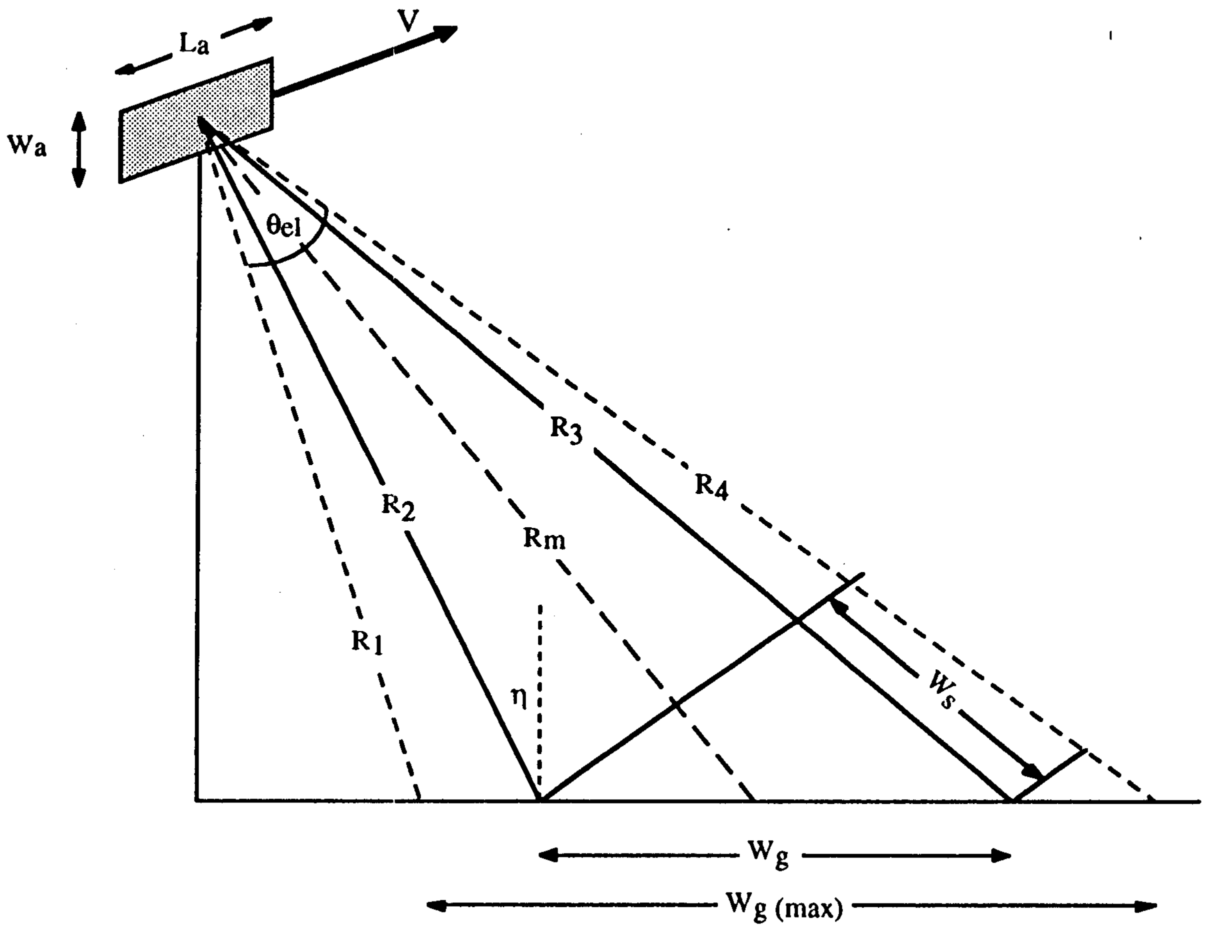

Figure 1: Simplified SAR geometry, showing the swath illuminated on the ground, $W_{g_{max}}$, which is determined by the antenna beamwidth, θ_{el}; and the actual swath for which returns are recorded, W_g, which is determined by the radar designer. In slant range the illuminated swath lies between R_1 and R_4, and the recorded swath lies between R_2 and R_3.